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ORIGINAL PAPER

Urban-rural migration, unemployment, and privatization: a synthesis of Harris-Todaro model and a mixed duopoly

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Abstract The purpose of this paper is to analyze the effect of privatization on unemployment under the dualistic economy where the manufactured goods sector in urban area is the mixed duopoly and the agricultural goods sector in rural area is competitive. The result shows that the privatization of public firm in urban area makes worse the urban unemployment.The partial privatization is optimal when the environmental damage is high, relatively.

Keywords Mixed duopoly · Privatization · Urban unemployment · Environmental pollution

JEL Classification L32 · Q56 · R11

1 Introduction

The purpose of this paper is to analyze the effect of privatization of public firm on urban unemployment in a dualistic economy. Some developing countries face on not only urban unemployment but also environmental problems. These countries need a prescription to solve an environmental problem and the unemployment problem at the same time. As for urban unemployment problem, Harris and Todaro (1970) elucidated the mechanism to bring about the unemployment caused by migration between urban area and rural area in developing countries. Some researchers extend this model by introducing environmental factor into Harris and Todaro model. Fukuyama and Naito (2007) also consider the environmental policies in the framework of Harris

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and Todaro model, in which the manufactured goods sector in urban area faces on competitive market.

On the other hand, some developing countries have promoted the privatization of some public sector due to any economic policy. Some researchers have concerned about the privatization of public firm and analyzed the mixed oligopoly market. De Fraja and Delbono (1989) is one of pioneer studies in mixed oligopoly market. Matsumura (1998) or, Beladi and Chao (2006) consider the partial privatization of public firm in mixed oligopoly.

In this paper, we construct the model, which synthesize three factors; mixed oligopoly market, dualistic economy, and environmental factor. We analyze the effect of privatization of public firm on urban unemployment or optimal privatization level under this synthesized model. The paper proceeds as follows. The next section outlines the basic model. Section 3 shows the effect of privatization of public firm on the equilibrium and optimal privatization in the model. Finally, Sect. 4 contains our concluding remarks.

2 The model

We consider the economy where all households have a common preference and reside in either urban area or rural area. We consider two kinds of products: agricultural goods and manufactured goods. The labor is used as the factors of production for both the agricultural good and the manufactured good. We assume that the urban wage is fixed above the market-clearing level, and that it has downward rigidity attributable to the minimum wage system and so on. Particularly, let $\bar{w}_x$ $(= w_x)$ represent the minimum wage in the urban area, which is higher than the rural wage w_y.[1] That rural wage is determined in the labor market of the agricultural goods sector and is equal to the marginal product of labor in the agricultural goods sector. Moreover, we assume that labor has free mobility between urban area and rural areas, and we deal with the agricultural goods as numeraire in our model. We normalize total population in the economy as one. The profit of agricultural goods sector and aggregated land rent in rural area are redistributed to all households.

2.1 Households

Since we assume that all households have common preference, they have the following same utility function.

$$U_i(x, y; D) = ax - \frac{1}{2}x^2 + y - D(\bar{X}) \quad (i = x, u, y) \tag{1}$$

where x, y, and D are the consumption of manufactured goods, that of agricultural goods, and environmental damage caused by the production of manufactured good sector, respectively. Though the environmental damage D depends on the production level of manufactured goods $(\bar{X})$, the households deal with this environmental damage as given. Thus, households cannot control this level, though the environmental

[1] This assumption is followed by Harris and Todaro (1970).

damage affects the utility level of households. All households have a unit of labor and supply it to either manufactured goods sector or agricultural goods sector inelastically. Thus, households employed by manufactured goods sector in urban area earn the wage income w_x $(= \bar{w}_x)$ and those employed by agricultural goods sector in rural area earn the wage income w_y. Households residing in the urban area are unemployed if they are not hired by the manufacturing goods sector. They do not earn wage income $(w_u = 0)$ and can get only redistribution from the profit of agricultural goods sector and the aggregated land lent in rural area. Here the budget constraint of each household is as follows.

$$w_i + I = px + y \quad (i = x, u, y) \tag{2}$$

where p and I denote the price of manufactured goods and resitribution for each household. Here, I is composed of land rent revenue and the profit of agricultural goods sector. Maximizing (1) subject to each budget constraint (2), the demand function of manufactured goods of households is as follows.

$$x = a - p \tag{3}$$

Substituting (3) into each budget constraint, each household's demand of agricultural goods are as follows.

$$y_i = w_i + I - p(a - p) \quad (i = x, u, y) \tag{4}$$

Moreover, substituting (3) and (4) into (1), we can derive the following indirect utility function of each household. Let v_i $(i = x, u, y)$ represent the indirect utility function of households employed by manufactured goods sector in urban area, unemployed households in urban area, households in rural area, respectively.

$$v_i = \frac{1}{2}(a - p)^2 + w_i + I - D(\bar{X}) \quad (i = x, u, y) \tag{5}$$

After deriving (3) as the demand function of manufactured goods of each household, we consider the total demand of manufactured goods in the economy. Let L_x, L_y, and L_u represent the numbers of households employed by manufactured goods sector, households employed by agricultural goods sector, and unemployed households, respectively. Assuming that the population in the economy is normalized to one, the (inverse) demand function of manufactured goods in an economy is as follows.

$$p = a - \bar{X} \tag{6}$$

Similar to Harris and Todaro model, the migration equilibrium between urban area and rural area is established when the expected utility level in urban area is equal to the utility level in rural area. Now we define λ as the urban unemployment rate.

$$\lambda \equiv \frac{L_u}{L_x + L_u} \tag{7}$$

Thus, the migration equilibrium condition (Harris and Todaro condition) is given by

$$(1 - \lambda)v_x + \lambda v_u = v_y \quad \Longleftrightarrow \quad (1 - \lambda)\bar{w}_x = w \tag{8}$$

Assuming that total population in the economy is one, the population constraint is as follows.

$$L_x + L_u + L_y = 1 \tag{9}$$

Plugging (7) and (9), the population constraint is rewritten as follows.

$$L_x + (1-\lambda)L_y = (1-\lambda) \tag{10}$$

2.2 Production

2.2.1 Agricultural goods sector

We assume that labor and land are required for the production of agricultural goods. We assume that $\bar{S}_y$ is the fixed land input and that it is normalized land input to one. We set up the production function as follows.

$$Y = (L_y)^{\sigma}(\bar{S}_y)^{1-\sigma}, \quad \sigma \in (0,1) \tag{11}$$

Since we deal with agricultural goods as numeraire and assume that agricultural goods market is competitive, the wage rate of household employed by agricultural goods sector is equal to marginal product of product in its sector.

$$w = \sigma(L_y)^{\sigma-1} \tag{12}$$

2.2.2 Manufactured goods sector

Next we construct the model of manufactured goods sector. Here, labor is only input factor for manufactured goods production. Though the agricultural goods sector market is competitive, the manufactured good sector faces the mixed duopoly, in which public firm and private firm exist in the same market. Manufactured goods are produced in domestic country and are consumed there. Thus, we do not consider both import and export of manufactured goods in this model. Moreover, the public firm is owned by domestic government and maximize the domestic social welfare. On the other hand, the purpose of private firm is maximization of his profit and it is owned by the foreign country entrepreneur. We assume that the minimum wage rate in urban area is $\bar{w}_x$ as well as Harris and Todaro (1970). We label the public firm and private firm as firm 0 and firm 1, respectively. Now we assume that the production function of manufactured goods is a linear function with respect to labor. Thus, we describe the production function of firm i $(i = 0, 1)$ as follows.[2]

$$X_i = \frac{L_x^i}{m}, \quad m > 0 \ (i = 0, 1) \tag{13}$$

Here taking account of labor as only input of manufactured goods production and the shape of (13), the marginal cost C_i of firm i is given by

$$C_i = \bar{w}_x m X_i \tag{14}$$

We assume that the environmental damage is caused by pollution, which the manufactured goods sector emits in the process of his production. Let $\bar{X}$ represent the total amount of manufactured goods produced by public firm and private firm, that is,

[2] See Daitoh (2010).

$\bar{X} = X_0 + X_1$. We set up the environmental damage caused by manufactured goods production as follows.

$$D(\bar{X}) = \frac{d}{2}(X_0 + X_1)^2 \quad (0 < d < 1) \tag{15}$$

Moreover we define $\hat{W}$ as the domestic social welfare function. Since the private firm is owned by foreign country entrepreneur in our model, the social welfare function does not include the profit of private firm. Thus the social welfare function is composed of consumer surplus, the profit of public firm, and environmental damage.[3]

$$\hat{W} = \frac{1}{2}(\bar{X})^2 + \pi_0 + \bar{w}_x L_x + w L_y - D(\bar{X}) + I$$

Here we consider that the government owns the share of $(1-\theta)$ of public firm as well as Matsumura (1998). Thus, the purpose of public firm (firm 0) is to maximize the weighted average of social welfare and its profit, which is defined by $V(\theta)$. It is possible for us to take account of partial privatization as to optimal privatization level.

$$\begin{aligned} V(\theta) &= \theta\pi_0 + (1-\theta)\hat{W} \\ &= \pi_0 + (1-\theta)\left\{\frac{1-d}{2}(X_0 + X_1)^2 + m\bar{w}_x(X_0 + X_1) + \sigma L_y^{\sigma} + I\right\} \end{aligned} \tag{16}$$

The profit function of firm 1, π_1, is given by

$$\pi_1 = \big[a - (X_0 + X_1)\big]X_1 - \bar{w}_x m X_1 \tag{17}$$

Thus, differentiating (17) with respect to X_1, the first order condition for profit maximization of firm 1 is as follows.

$$\frac{\partial \pi_1}{\partial X_1} = a - X_0 - 2X_1 - m\bar{w}_x = 0 \tag{18}$$

On the other hand, as the public firm maximizes (16), the first order condition to maximize the weighted average of social welfare and profit is as follows.

$$\frac{\partial V}{\partial X_0} = a - \theta m\bar{w}_x + \big[(1-\theta)(1-d) - 2\big]X_0 + \big[(1-\theta)(1-d) - 1\big]X_1 = 0 \tag{19}$$

Solving (18) and (19), the equilibrium production of each firm can be derived as X_0^* and X_1^*:

$$X_0^* = \frac{[1 + (1-\theta)(1-d)]a + [1 - 2\theta - (1-\theta)(1-d)]m\bar{w}_x}{3 - (1-\theta)(1-d)} \tag{20}$$

$$X_1^* = \frac{[1 - (1-\theta)(1-d)]a - [2 - \theta - (1-\theta)(1-d)]m\bar{w}_x}{3 - (1-\theta)(1-d)} \tag{21}$$

[3]The definition of social welfare used here is followed by Fukuyama and Naito (2007). Some countries like China do not permit firms owned by 100% foreign capital to operate in those countries. However, the Malaysian government permits 100% foreign capital to operate their firms in Malaysia after 2009. We consider the later situation in this model.

Comparing (20) with (21), we know that the public firm production is larger than the private firm production when the parameter d, which means the environmental damage caused by manufactured goods sector, satisfies $0 < d < 1$. This result is the same as the results derived in previous literatures about mixed oligopoly theory and the public firm's behavior outsells that of private firms.

$$\bar{X} = \frac{2a - (1+\theta)m\bar{w}_x}{3 - (1-\theta)(1-d)} \tag{22}$$

Here let L_x represent the labor demand of manufactured goods sector. Since we specify (13) as the production function of manufactured goods, the labor demand of manufactured goods sector in equilibrium is as follows.

$$L_x = L_x^0 + L_x^1 = \frac{m[2a - (1+\theta)m\bar{w}_x]}{3 - (1-\theta)(1-d)} \tag{23}$$

where L_x^i $(i = 0, 1)$ denotes the labor input of firm i $(= 1, 2)$. Substituting (22) into (6), we derive the equilibrium price of manufactured goods as follows.

$$p^* = \frac{a[1 - (1-\theta)(1-d)] + (1+\theta)m\bar{w}_x}{3 - (1-\theta)(1-d)} \tag{24}$$

Next we consider the effect of privatization of public firm on total production in manufactured goods sector. Differentiating (22) with respect to θ, we derive the following function.

$$\frac{\partial \bar{X}}{\partial \theta} = -\frac{(1-d)a + [3 - 2(1-\theta)(1-d)]m\bar{w}_x}{[3 - (1-\theta)(1-d)]^2} < 0 \tag{25}$$

Thus, we know that the effect of privatization on total production depends on the degree for environmental damage, which is denoted by d. Here we assume that d is positive and smaller than one. So the privatization of public firm leads to decrease total manufactured goods production. Moreover, since we assume the production function is linear with respect to labor, the effect of privatization on the required labor input in equilibrium is as follows.

$$\frac{\partial L_x}{\partial \theta} = -\frac{m\{(1-d)a + [3 - 2(1-\theta)(1-d)]m\bar{w}_x\}}{[3 - (1-\theta)(1-d)]^2} < 0 \tag{26}$$

The required labor input in manufactured goods sector depends on the degree for environmental damage, too. We derive the following lemma from (25) and (26).

Lemma 1 *The privatization of public firm decreases the total amount of manufactured goods production and the required labor input for its production.*

3 The effect of privatization on urban unemployment

We have specified the behaviors of households, agricultural goods sector, and manufactured goods sector. We consider the migration between urban area and rural area

under the above specification. The wage rate in rural area is given by (12). So substituting (12) into Harris-Todaro condition (8), L_y is derived as follows.

$$L_y^* = \left(\frac{(1-\lambda)\bar{w}_x}{\sigma}\right)^{\frac{1}{\sigma-1}} \tag{27}$$

Moreover, substituting (23) and (27) into (10), we derive the following equation.

$$\frac{m[2a-(1+\theta)m\bar{w}_x]}{3-(1-\theta)(1-d)} + (1-\lambda)^{\frac{\sigma}{\sigma-1}}\left(\frac{\bar{w}_x}{\sigma}\right)^{\frac{1}{\sigma-1}} = (1-\lambda) \tag{28}$$

Solving (28) with respect to λ, we can describe λ as the function of $\bar{w}_x, \theta, m, d$, and σ. Using the implicit function theorem to (28), $\frac{d\lambda}{d\theta}$ is given by

$$\frac{d\lambda}{d\theta} = \frac{m[2(1-d)a+(1-\theta)dm\bar{w}_x]}{[(\frac{\sigma}{1-\sigma})(1-\lambda)^{\frac{1}{\sigma-1}}(\frac{\bar{w}_x}{\sigma})^{\frac{1}{\sigma-1}}+1][3-(1-\theta)(1-d)]^2} > 0 \tag{29}$$

We know that the effect of privatization of public firm on urban unemployment depends on the degree for environmental damage caused by manufactured good sector. Since $1-d$ is positive, the sign of $\frac{d\lambda}{d\theta}$ is positive. Thus, we derive the following proposition.

Proposition 1 *When the degree of environmental damage caused by manufactured goods production is smaller than one, the privatization of public firm makes worse the unemployment rate in urban area.*

We show that it is possible to make worse the unemployment rate by the privatization of public firm under any degree of environmental damage due to Proposition 1. Next we consider how the privatization affects the environmental damage in an economy. As we know from (15), the increase of manufactured goods production leads to increase the environmental damage in an economy. We consider the effect of privatization on total manufactured goods production in equilibrium. Differentiating $D(X(\theta))$ with respect to θ, we know the following relationship from (25).

$$\frac{dD(\bar{X})}{d\theta} = \frac{dD(\bar{X})}{d\bar{X}} \cdot \frac{d\bar{X}}{d\theta} < 0 \tag{30}$$

Therefore, the effect of public firm's privatization on environmental damage depends on the sign of (25). Supposed that d is smaller than one, $\frac{d\bar{X}}{d\theta}$ is negative, that is, the privatization improves the environmental damage. The public firm determines his production level to maximize the social welfare and produces more excessive than the private firm under the mixed oligopoly model. Since the privatization of public firm affects the unemployment rate and environmental damage, each effect of privatization is opposite. Thus, it is necessary to analyze the effect of privatization on social welfare in equilibrium in order to derive the optimal privatization level of

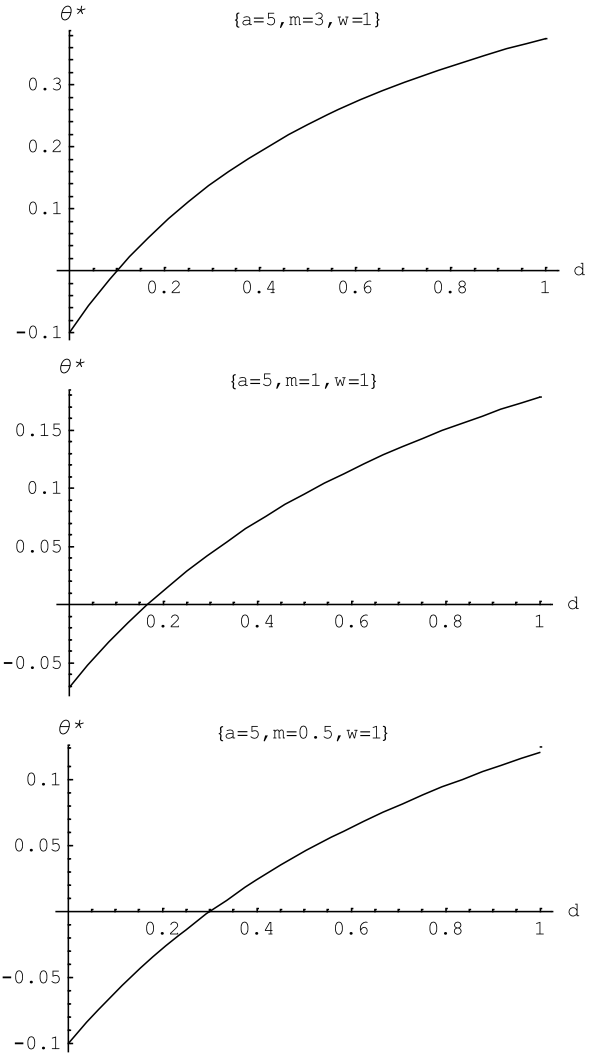

Fig. 1 Optimal privatization and environmental damage

public firm. Since equilibrium output of public firm is determined by (20), the social welfare in equilibrium is as follows.

$$\bar{W}^* = \frac{1-d}{2}(\bar{X})^2 + p^* X_0^* + m\bar{w}_x L_x^* + wL_y^* + I \tag{31}$$

Differentiating $\bar{W}^*$ with respect to θ, we derive the following equation.

$$\begin{aligned}\frac{\partial \bar{W}^*}{\partial \theta} = &-\frac{1}{(2+\theta+(1-\theta)d)^3}\big[2(1-d)a + (1+2d)m\bar{w}_x\big] \\ &\times\big[\big(3(1+d)\theta - d\big)a + \big(2-m+\theta-2m(\theta+d)+(1-\theta)d\big)\big]\end{aligned} \tag{32}$$

Determining θ^* to satisfy $\frac{\partial \bar{W}^*}{\partial \theta} = 0$, the optimal privatization level is given as the function of d.[4]

$$\theta^* = \frac{(a + 2m - 1)d + m - 2}{(3a - 1)d + (3a - 2m + 1)} \tag{33}$$

The sign of (32) depends on the environmental damage parameter d and the productivity of manufactured goods m. Now we consider about relationship between optimal privatization level and the environmental damage parameter d.

Figure 1 describes the relationship between optimal privatization level θ^* and the environmental damage parameter d under each parameter of manufactured goods productivity. As we know from Fig. 1, the optimal privatization level is the increasing function of environmental damage parameter. Since we assume that the range of parameter d is $d \in [0, 1]$, $\theta^* = 1$ (full privatization) is not optimal despite the productivity of manufactured goods. On the other hand, it is possible that $\theta^* = 0$ (full nationalization) is optimal when the environmental damage parameter d is low relatively. Moreover, the range of environmental damage parameter, where full nationalization is optimal, is the decreasing function of the productivity of manufactured goods. In other words, the partial privatization is optimal when the environmental damage is high relatively.

4 Concluding remarks

We construct the model introducing the environmental damage having an effect on household's utility into Harris and Todaro model, which describes a dualistic economy in developing countries. Though Fukuyama and Naito (2007) introduced the environmental damage into Harris and Todaro model and assumed that both manufactured goods market and agricultural goods market are competitive, we consider the manufactured good market is mixed duopoly market. The results show that the privatization of public firm always makes worse urban unemployment rate and that the partial privatization is optimal when the productivity of manufactured goods and environmental damage is high relatively.

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