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# Can urban pollution shrink rural districts?

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**Abstract** This paper discusses how the externality of environmental damage affects the equilibrium properties of a simple overlapping generations model with multiple regions. Simulation results indicate that the environmental policy of the government decreases capital accumulation. When the government imposes an environmental tax on the urban sector, the urban-to-rural population ratio decreases, whereas the total fertility rate increases.

**Keywords** Environmental policy · Capital accumulation · Fertility · Population distribution

**JEL Classification** E61 · J13 · O52 · R23

## 1 Introduction

Acid rain caused by the urban sector has damaged productivity in the rural sector in Europe. In the developing countries such as India or China, air pollution and water

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pollution generated in urban areas have recently reduced productivity in rural areas. In selecting a residence, the conditions surrounding the prospective residence are important considerations. Thus, one of the objectives of this paper is to describe the economic situation of the residential district by constructing a dynamic spatial model. In examining the relationships between environment and regional migration, Hosoe and Naito (2006) analyze the effect of trans boundary pollution on regional distribution in a core–periphery model. Moreover, Fukuyama and Naito (2007) introduce environmental factor caused by urban sector into Harris and Todaro (1970) and analyze the effect of environmental policy on urban unemployment. However, these studies are analyzed with a static model and do not discuss the matt er from a dynamic perspective. Urban-rural migration must be considered using a dynamic model because environmental issues arise not only between regions but also between generations.

This paper examines how the environmental policy of the government dynamically affects the population distribution between an urban area and a rural area by intergenerational transfers. We introduce environmental externality into Yakita (2011), which combines intergenerational transfer issues with interregional migration. We examine the effects of environmental policy on urban-rural migration under the overlapping generations model.

The remainder of the paper is organized as follows. Section 2 presents the model and discusses the properties of market equilibrium. Section 3 clarifies the effects of environmental policy on capital accumulation, population distribution, and fertility based on Sect. 2. Finally, Sect. 4 presents concluding remarks.

## 2 Model

We consider a simple overlapping generations model and construct our model by introducing environmental externality into the Yakita (2011). The economy comprises identical three-period-lived agents, perfectly competitive firms, and a government.

### 2.1 Consumers

The economy examined in this paper consists of urban and rural regions. Household members reside and work in either urban area ($u$) or rural area($r$). We presume that individuals live in three periods: young, working, and the retired generations. The young generation consists of individuals raised by their parents. The working generation provides inelastic labor supply to firms in each period and allocate wage income to savings for consumption in the retirement period and expenses related to raising a child.[1] Finally, the retired generation comprises individuals who consume their accumulated savings.

[1] Taking child raising cost and education costs into consideration, Zhang and Zhang (1998) and Omori (2009) examines the effects of public policy on fertility.

The working generation at period $t$ is called generation $t$. Following Galor and Weil (1996), Kimura and Daishin (2007), and Yakita (2011), among others, the preference of a representative agent of generation $t$ in region $i(=u, r)$, $u_t^i$, is

$$u_t^i = \gamma \ln n_t^i + (1-\gamma) \ln c_{t+1}^i, \quad (i = u, r), \tag{1}$$

where $n_t^i$ denotes the number of children in period $t$, $c_{t+1}^i$ represents the consumption in period $t+1$ and $0 < \gamma < 1$.[2] Let $N_t$ be the total population of the working generation at period $t$ and $N_t^i$ be the population of the working generation at period $t$ in region $i$.[3] Therefore, we obtain $N_{t+1} = N_t^u n_t^u + N_t^r n_t^r$. In addition, $r_{t+1}$ is the interest rate at $t+1$, $s_t^i$ is the savings of the working generation in region $i$ at period $t$ and $w_t^i$ is the wage rate at $t$ in region $i(=u, r)$. The budget constraint of the working period for generation $t$ in region $u$ is expressed as $\left(1-\sigma - zn_t^u\right) w_t^u = s_t^u$, where $\sigma$ and $z$, respectively, represent the living cost in the urban area and the cost of having a child.[4] The budget constraint of the retirement period for generation $t$ in regions $u$ is $(1+r_t)s_t^u = c_{t+1}^u$. Therefore, the lifetime budget constraint of generation $t$ in regions $u$ is given by

$$\left(1-\sigma - zn_t^u\right) w_t^u = c_{t+1}^u. \tag{2}$$

Similarly, the budget constraint of the working period for generation $t$ in regions $r$ is $\left(1 - zn_t^r\right) w_t^r = s_t^r$. The budget constraint of the retirement period is shown by $(1+r_t)s_t^r = c_{t+1}^r$, and the lifetime budget in region $r$ is

$$\left(1 - zn_t^r\right) w_t^r = c_{t+1}^r \tag{3}$$

Maximizing the utility function in each region, (1), subject to either the budget constraint (2) or (3), we derive the optimal plans of individual in each region as

$$n_t^u = \frac{\gamma(1-\sigma)}{z}, \tag{4}$$
$$c_{t+1}^u = (1-\gamma)(1-\sigma)(1+r_{t+1}) w_t^u, \tag{5}$$
$$n_t^r = \frac{\gamma}{z}, \tag{6}$$

[2] Galor and Weil (1996) develop a growth model including the household's fertility/labor-supply choice and discuss the effects of fertility on economy. Galor and Weil (1996) assume such utility function as (1) without the working generation's consumption. Subsequently, Kimura and Daishin (2007) and Yakita (2011) also examine these effects with the same utility function. In the proposed model, we suppose the household's fertility/labor-supply choice and the number of children affects the labor supply. To simplify the discussion on the basis of these studies, we assume that the working generations derive the utility from the number of children and utility function, (1) is assumed without the factor of the consumption of the working generation. However, if we introduce this consumption of the working generation into a utility function, we can obtain the similar results.

[3] At initial period, we assume the positive population. That is, $N_0 > 0$ and $N_i > 0$.

[4] For example, we can interpret $\sigma$ as the congestion cost in urban area. Supposing the representative agent model in this paper, such cost is assumed to be identical between agents.

and

$$c^r_{t+1} = (1-\gamma)(1+r_{t+1})\, w^r_t. \tag{7}$$

We note that if $\sigma \to 0$, $n^u_t$ approaches $n^r_t t$.

### 2.2 Firms

We assume that technologies in urban and rural sectors are heterogeneous and that consumer goods are numeraire. Despite labor and capital being necessary for production in an urban area, only labor is required as input in a rural area. We also assume that productivity in the rural area, $\Omega$, depends on the environmental damage caused by the urban sector, which is affected by the capital input in the urban sector and beyond the control of the rural sector.[5] The aggregate output, $Y_t$, is produced by the capital input at period $t$, $K_t$, and the labor supply in area $i(=u, r)$ at period $t$, $L^i_t$.[6] We specify the aggregate production function as

$$Y_t = A\,(K_t)^{\alpha}\left(L^u_t\right)^{1-\alpha} + \Omega L^r_t, \quad (0 < \alpha < 1), \tag{8}$$

where $A$ and $\Omega$ are productivity of the urban sector and the productivity of the rural sector, respectively. We assume that the government imposes environmental tax $\tau$ on output in the urban area. Solving the profit maximization problem of the urban sector and the rural sector, the wage rates, respectively, in both regions are derived as

$$w^u_t = A(1-\tau)(1-\alpha)(K_t)^{\alpha}\left(L^u_t\right)^{-\alpha} = A(1-\tau)(1-\alpha)\left(\frac{K_t}{L^u_t}\right)^{\alpha} \tag{9}$$

and

$$w^r_t = \Omega. \tag{10}$$

In addition, we can express the interest rate as $1 + r_t = A(1-\tau)\alpha\,(K_t)^{\alpha-1}\left(L^u_t\right)^{1-\alpha}$.

### 2.3 Environmental quality

We consider that production in the urban sector emits pollution. The pollution level depends on the capital input of the urban production function because factories are the sources of pollution, and the scale of factories affects the pollution level in the rural area.[7] Following Copeland and Taylor (1999), we suppose that the pollution emitted in an urban area influences productivity in the rural sector; that is, we regard

[5] This setting is similar to that described by Fukuyama and Naito (2007).

[6] At the initial period, we assume the capital to be positive; that is, $K_0 > 0$.

[7] When capital accumulation is shown by the number of factory, as the number of factory increases, the pollution worsens.

the productivity function of the rural sector as a function of environmental quality depending on the capital input in the urban area and the environmental policy of the government. We define the productivity function of the rural area in (8) as

$$\Omega \equiv b \times \frac{g}{k_t^{\delta}}, \tag{11}$$

where $b$ represents the positive parameter, $g$ denotes the government environmental improvement per capita, $k_t$ is the capital per capita ($k_t \equiv \frac{K_t}{N_t}$) at period $t$, and $\delta$ is the scale parameter for $k_t$.[8]

### 2.4 Government

The government is assumed to behave under a balanced budget regime. It collects the output tax of firms in the urban area and finances the environmental improvement per capita, $g$, in the current period. That is,

$$A\tau \left(K_t\right)^{\alpha} \left(L_t^u\right)^{1-\alpha} = gN_t, \tag{12}$$

where $\tau$ is the output tax rate.

### 2.5 Equilibrium

Let $\phi_t$ denote the ratio of the urban population to the total population; that is, $\phi_t = \frac{N_t^u}{N_t}$. The total fertility rate at period $t$, $m_t$, can be written as

$$m_t = \phi_t n_t^u + (1-\phi_t)\, n_t^r. \tag{13}$$

We can show the labor supply in each area, respectively, as

$$L_t^u = N_t \phi_t \left(1-\sigma - z n_t^u\right), \tag{14}$$

and

$$L_t^r = N_t \left(1-\phi_t\right) \left(1 - z n_t^r\right). \tag{15}$$

Labor supply in the urban area, (14), is affected by congestion costs because this cost is considered in fertility/labor supply choice. Then, from (12), by using (4) and (14), the government's per capita budget constraint can be rewritten as

$$\tau A \left[\phi_t (1-\sigma)(1-\gamma)\right]^{1-\alpha} k_t^{\alpha} = g. \tag{16}$$

[8] In both areas, the consumers who save for future consumption are the suppliers of capital. Therefore, the capital per capita is the aggregate capital divided by the total population.

In equilibrium, the representative indirect utility function in each area, $V_t^i$ can be shown as

$$V_t^u = \ln \left[ \frac{\gamma\,(1-\sigma)}{z} \right]^{\gamma} \left[ (1-\gamma)\,(1+r_{t+1})\, w_t^u\,(1-\sigma) \right]^{(1-\gamma)}, \tag{17}$$

and

$$V_t^r = \ln \left[ \frac{\gamma}{z} \right]^{\gamma} \left[ (1-\gamma)\,(1+r_{t+1})\, w_t^r \right]^{(1-\gamma)}. \tag{18}$$

Given that each household has no incentive to migrate between regions in equilibrium, each household enjoys the same equilibrium utility level regardless of the difference in wage between regions. When each indirect utility is equalized across the regions in equilibrium, $V_t^u = V_t^r$, we can obtain the interregional equilibrium condition, $(1-\sigma)(w_t^u)^{1-\gamma} = (w_t^r)^{1-\gamma}$. By using (4), (9), (10), (11), (14), and (16), this condition is simplified as

$$\tilde{k}_t = \left[ \frac{b\tau\phi_t(1-\gamma)}{(1-\tau)(1-\alpha)(1-\sigma)^{\frac{\gamma}{1-\gamma}}} \right]^{\frac{1}{\delta}}, \tag{19}$$

where $\tilde{k}_t$ is the capital per-capita threshold that corresponds to the complete depopulation of the rural area, $\phi_t = 1$.[9] We note that the sign of differentiation in (19) with respect to $\tau$ is always positive.That is, from (19),

$$\frac{d\tilde{k}_t}{d\tau} = \frac{1}{\sigma} \left[ \frac{b\tau\phi_t(1-\gamma)}{(1-\tau)(1-\alpha)(1-\sigma)^{\frac{\gamma}{1-\gamma}}} \right]^{\frac{1}{\delta}-1} \left[ \frac{b\phi_t\left(1+\frac{\tau}{1-\tau}\right)}{(1-\tau)(1-\alpha)(1-\sigma)^{\frac{\gamma}{1-\gamma}}} \right] > 0.$$

Increasing tax rate rises up the wage rate in rural area, $w_t^r$, through changing $g$ and $\Omega$. Because $w_t^r$ increases $s_t^r$ and $K_t$, $\frac{d\tilde{k}_t}{d\tau}$ is positive.

From (19), the ratio of urban population to the total population, $\phi_t$ is

$$\phi_t = \begin{cases} 1 & k_t \geq \tilde{k}, \\ \theta k_t^{\delta} & k_t < \tilde{k}, \end{cases} \tag{20}$$

where

$$\theta = \frac{(1-\tau)(1-\alpha)(1-\sigma)^{\frac{\gamma}{1-\gamma}}}{\tau b(1-\gamma)}. \tag{21}$$

[9] If the capital intensity in the urban sector $k_t$ exceeds $\tilde{k}_t$, we have $\phi_t = 1$. In this case, all households reside in the urban area, and no household resides in the rural area.

Derived from (20), the total fertility rate, $m_t$, can be rewritten as

$$m_t = \begin{cases} \frac{\gamma(1-\sigma)}{z} & k_t \geq \tilde{k}\,(\phi_t = 1)\,, \\ \frac{\gamma\left(1-\sigma\theta k_t^{\delta}\right)}{z} & k_t < \tilde{k}\left(\phi_t = \theta k_t^{\delta}\right). \end{cases} \tag{22}$$

Finally, the market-clearing condition for capital markets is expressed as $K_{t+1} = N_t^u s_t^u + N_t^r s_t^r$. Given (9), (10), (14), (15), (20), and (22), the market-clearing condition is written as

$$k_{t+1} = \begin{cases} \left(\frac{Az}{\gamma}\right)(1-\gamma)^{1-\alpha}(1-\tau)(1-\alpha)(1-\sigma)^{-\alpha}(k_t)^{\alpha}, & k_t \geq \tilde{k}\,(\phi_t = 1)\,, \\ \frac{z}{\gamma\left[1-\sigma\theta(k_t)^{\delta}\right]} \\ \times\left\{\theta^{1-\alpha}(1-\gamma)^{1-\alpha}(1-\sigma)^{1-\alpha} A\,(1-\tau)\,(1-\alpha)\,k_t^{\alpha+(1-\alpha)\delta}\right. \\ \left.+bg((1-\gamma)\,k_t^{-\delta} - \theta\,(1-\gamma))\right\}, & k_t < \tilde{k}\left(\phi_t = \theta k_t^{\delta}\right). \end{cases} \tag{23}$$

On (23), when $k_t$ increases and excceds to $\tilde{k_t}$, the path of $k_{t+1}$ shifts to the path of $\phi_t = 1$. In other words, when $k_t < \tilde{k_t}$, the path of $k_{t+1}$ is on the path of $\phi_t = \theta k_t^{\delta}$.

Considering that this paper is aimed at investigating policy effects on the population distribution between the urban and the rural areas, we do not specifically examine the economy when $k_t \leq \tilde{k}\,(\phi_t = 1)$, where the population of the rural area is zero. Instead, we specifically examine the economy when $k_t > \tilde{k}\left(\phi_t = \theta k_t^{\delta}\right)$.

Differentiating (23) with respect to $k_t$ in the neighborhood of equilibrium, we derive the following equation:

$$\begin{aligned} \frac{dk_{t+1}}{dk_t} = & \frac{Mk_t^{-1}}{\left[\gamma\left(1-\sigma\theta k_t^{\delta}\right)\right]^2} \\ & \times\left\{\gamma\left(1-\sigma\theta k_t^{\delta}\right)\left[(\alpha+(1-\alpha)\,\delta)\left[(1-\tau)\,(1-\alpha)-b\,(1-\gamma)\,\tau\theta\right]k_t^{\alpha+(1-\alpha)\delta}\right.\right. \\ & \left. + b\,(1-\gamma)\,\tau k_t^{\alpha(1-\delta)}\right] \\ & \left. +\left[\left[(1-\tau)\,(1-\alpha)-b\,(1-\gamma)\,\tau\theta\right]k_t^{\alpha+(1-\alpha)\delta}+b\,(1-\gamma)\,\tau k_t^{\alpha(1-\delta)}\right]\gamma\delta\sigma\theta k_t^{\delta}\right\}, \end{aligned} \tag{24}$$

where $M = z\theta^{1-\alpha}(1-\gamma)^{1-\alpha}(1-\sigma)^{1-\alpha}A$. When $(1-\tau)\,(1-\alpha) - b\,(1-\gamma)\,\tau\theta$ is positive, the sign of $\frac{dk_{t+1}}{dk_t}$ is positive. In addition, we find that (23) is concave when $k_t \geq \tilde{k}\,(\phi = 1)$. However, (23) is either concave or convex depending on each parameter when $k_t < \tilde{k}\left(\phi_t = \theta k_t^{\delta}\right)$. If (23) is convex, multiple equilibria can be exist under some parameters in this model. In the study by Yakita (2011), $k_{t+1}$ is always convex with respect to $k_t$ when $k_t \geq \tilde{k}$. Given the introduction of environmental externality to the study by Yakita (2011), $k_{t+1}$ might not be convex. However, with qualitative analysis, we are difficult to show the dynamic path of $k_{t+1}$ and to examine the stability on equilibrium in this paper. In next section, the simulation analysis allows us to do such discussions.

## 3 Effects of government policy

In the last section, we introduce environmental externality to Yakita (2011). Given that the model exclude environmental externality, environmental policy is briefly discussed. However, public environmental policy can be discussed in detail because we incorporated the environmental factor into the model. In the proposed model, we imposed tax on urban sector and use tax revenue to improve productivity in the rural sector. Consequently, examining the effect of environmental tax on equilibrium allows us to discuss the policy effects on the steady state. For the following analysis, we can rewrite the capital at the steady state, $\hat{k}$, as

$$\hat{k} = \frac{M}{\gamma\left(1-\sigma\theta\hat{k}^{\delta}\right)} \times \left[\left[(1-\tau)(1-\alpha) - b(1-\gamma)\tau\theta\right]\theta^{1-\alpha}\hat{k}^{\alpha+(1-\alpha)\delta} + \theta^{1-\alpha} b(1-\gamma)\tau\hat{k}^{\alpha(1-\delta)}\right] \tag{25}$$

Thus, we describe $\hat{k}$ at the steady state in Fig. 1 under adequate parameters to analyze the effect of environmental policy parameter, $\tau$, on $\hat{k}$. Stability can be discussed in this model because of the possible occurrence of multiple equilibria. With adequate parameters, our simulations demonstrate equilibrium in the proposed model. We adopt the

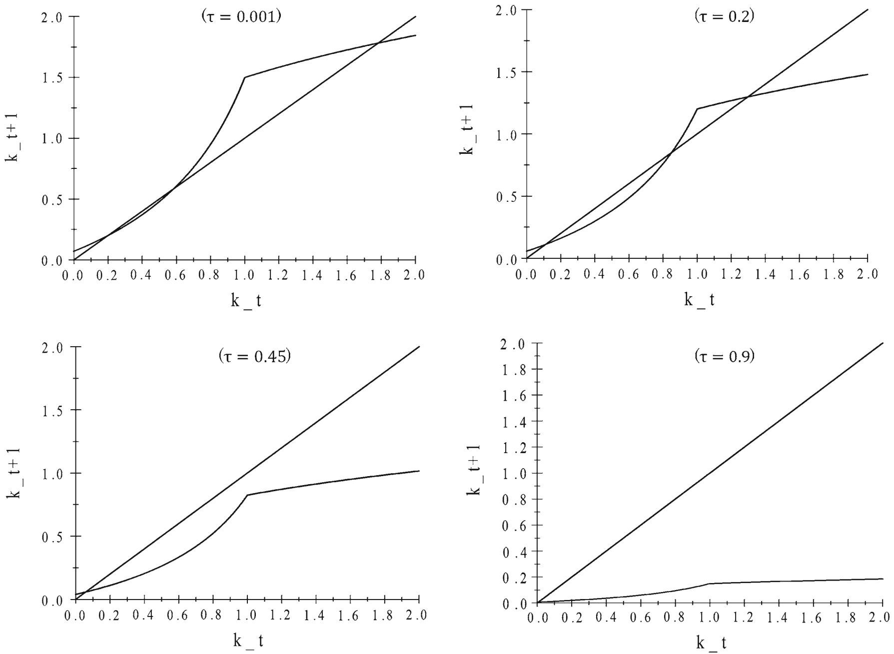


**Fig. 1** The effect of $\tau$ on steady state

parameters, except for $\tau$, as follows: $\delta = 1, \sigma = 0.6, z = 0.1, \alpha = 0.3, \gamma = 0.7, A = 50, b = 0.132$ ($\theta = 1$). We use $\tau = 0.001, 0.2$, and 0.45 in the simulation analysis.

Figure 1 shows that the number of equilibrium depends on the value of $\tau$. The number of stable equilibrium is 2 when $\tau$ is relatively small. However, the equilibrium is unique when the $\tau$ is relatively large. Additionally, in some figures, on the neighborhood in the crosspoints of the path of $k_{t+1}$ and 45 degree line, the path of $k_{t+1}$ is concave. Under some parameters, we find that the equilibrium is stable. However, under other parameters, the equilibrium can not be existed in this model. In other words, for the existence of eqilibrium, the introduction of government sector into model should be needed. We also find that the reinforcement of environmental policy reduces the stable steady state $\hat{k}$ because an increase in $\tau$ leads to a decrease in wages in urban areas.[10]

From (20), given that the ratio of urban population to total population, $\phi_t$, is the increasing function of $k$ if $k < \tilde{k}$, the reinforcement of environmental policy tends to decrease the population in the urban area.[11] That is, with the combination of this simulation and the discussion in Appendix A, the effects of $\tau$ on the ratio of the urban to total population at the steady state, $\hat{\phi}$ is negative $\left(\frac{d\hat{\phi}}{d\hat{k}}\frac{d\hat{k}}{d\tau} < 0\right)$. A higher tax contributes to the environmental quality and the wage rates in rural areas. Such a tax promotes immigration from the urban to the rural area.

Similarly, given the effects of $\tau$ on total fertility rate at the steady state, $\hat{m}$, is positive, the urban wage decreases but the rural wage increases. In the rural area where wage increases and the cost of raising children decreases, a higher tax rate enhances the fertility rate. These findings lead to the following proposition:

**Proposition 1** *If $\hat{k} < \tilde{k}$ in the steady state, $\frac{d\hat{k}}{d\tau}$ is negative, $\frac{d\hat{\phi}}{d\tau}$ is negative, and $\frac{d\hat{m}}{d\tau}$ is positive.*

## 4 Concluding remarks

As described in this paper, we analyzed how the government environmental policy affects the population distribution between the urban and rural areas in a dynamic model. We showed that imposing a government environmental policy leads to less accumulation of capital. Such a policy also decreases the ratio of the urban population to the rural population but increases the total fertility rate. To recover from urban pollution and rural decline, the government should adopt not only direct regulations but other environmental policies as well.

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[10] The condition for stability at the steady state is that the slope of (23) must be smaller than 1.

[11] The comparative statics on equilibrium appears in Appendix A.

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## Appendix A: Comparative statics of (25)

Applying the implicit function theorem to (25) obtains

$$\frac{d\hat{k}}{d\tau} = \frac{\Phi}{\Psi}, \tag{26}$$

where

$$\begin{aligned}\Phi =& \gamma\left(1-\sigma\theta\hat{k}^{\delta}\right)\Big[\theta^{1-\alpha}\hat{k}^{\alpha+(1-\alpha)\delta}\Big(-(1-\alpha)-b(1-\gamma)\theta^{-1-\alpha}\\ &+(2-\alpha)b(1-\gamma)\tau\frac{\partial\theta}{\partial\tau}\Big)+\hat{k}^{\alpha(1-\delta)}\theta^{-\alpha}\left((1-\alpha)\frac{\partial\theta}{\partial\tau}b(1-\gamma)\tau+\theta b(1-\gamma)\right)\Big]\\ &+\left[((1-\tau)(1-\alpha)-b(1-\gamma)\tau\theta)\theta^{1-\alpha}\hat{k}^{\alpha+(1-\alpha)\delta}+\theta^{1-\alpha}b(1-\gamma)\tau\hat{k}^{\alpha(1-\delta)}\right]\sigma\frac{\partial\theta}{\partial\tau}\hat{k}^{\delta},\end{aligned} \tag{27}$$

and

$$\begin{aligned}\Psi =& \frac{\left[\gamma\left(1-\sigma\theta\hat{k}^{\delta}\right)\right]^2}{M}-\gamma\left(1-\sigma\theta\hat{k}^{\delta}\right)\Big[((1-\tau)(1-\alpha)-b(1-\gamma)\tau\theta)\theta^{1-\alpha}\\ &(\alpha+(1-\alpha)\delta)\hat{k}^{\alpha+(1-\alpha)\delta-1}+\alpha(1-\delta)\theta^{1-\alpha}b(1-\gamma)\tau\hat{k}^{\alpha(1-\delta)-1}\Big]\\ &-\left[((1-\tau)(1-\alpha)-b(1-\gamma)\tau\theta)\theta^{1-\alpha}\hat{k}^{\alpha+(1-\alpha)\delta}+\theta^{1-\alpha}b(1-\gamma)\tau\hat{k}^{\alpha(1-\delta)}\right]\sigma\theta\delta\hat{k}^{\delta-1}.\end{aligned} \tag{28}$$

The sign of (26) is ambiguous. Thus, we determine the sign of (26). From the numerator of (26), $\Phi$, as $(1-\tau)(1-\alpha)-b(1-\gamma)\tau\theta > 0$, if $\alpha$ is less than $\tau$, $\Phi$ is negative. On the other hand, on denominator of (26), $\Psi$, if

$$\begin{aligned}&\frac{\left[\gamma\left(1-\sigma\theta\hat{k}^{\delta}\right)\right]^2}{M} > \gamma\left(1-\sigma\theta\hat{k}^{\delta}\right)\Big[((1-\tau)(1-\alpha)-b(1-\gamma)\tau\theta)\theta^{1-\alpha}\\ &(\alpha+(1-\alpha)\delta)\hat{k}^{\alpha+(1-\alpha)\delta-1}+\alpha(1-\delta)\theta^{1-\alpha}b(1-\gamma)\tau\hat{k}^{\alpha(1-\delta)-1}\Big]\\ &+\left[((1-\tau)(1-\alpha)-b(1-\gamma)\tau\theta)\theta^{1-\alpha}\hat{k}^{\alpha+(1-\alpha)\delta}+\theta^{1-\alpha}b(1-\gamma)\tau\hat{k}^{\alpha(1-\delta)}\right]\sigma\theta\delta\hat{k}^{\delta-1},\end{aligned} \tag{29}$$

$\Psi$ is positive.

Government policy affects the steady-state capital either directly or indirectly. In the indirect effect, government policy influences capital by improving environmental quality. We can demonstrate the effects of tax on disposable income in (27), $\tau$, and the effects of tax on disposable income through changing government expenditure in (27), $\alpha$. When the effect of the latter exceeds that of the former, $\Phi$ is negative.

The direct efect by which such a policy affects saving behavior in both the urban and the rural areas is shown in (29). The left-han side of (29) shows how tax affects savings in the urban area. The right-hand side of (29) shows the effects of tax on savings in the rural area. When the effect of the former is weaker than that of the latter, the denominator of (26), $\Psi$ is positive. Therefore, if these conditions are satisfied, $\frac{d\hat{k}}{d\tau}$, is negative.

We subsequently examine the ratio of the urban to the total population, $\hat{\phi}$, and the total fertility rate, $\hat{m}$, in a steady state. The effect of $\tau$ on $\hat{\phi}$ is negative as $\frac{d\hat{\phi}}{d\tau} = \frac{d\theta}{d\tau}\hat{k}^{\delta} + \theta\delta\hat{k}^{\delta-1}\frac{d\hat{k}}{d\tau} < 0$ from (20). A higher tax contributes to the environmental quality and the wage rates in the rural area. This tax promotes immigration from urban to rural. Furthermore, the effect of $\tau$ on $\hat{m}$ is positive by $\frac{d\hat{m}}{d\tau} = \frac{-\gamma Z\sigma\left(\hat{k}^{\delta}\frac{d\theta}{d\tau} + \delta\hat{k}^{\delta-1}\frac{d\hat{k}}{d\tau}\right)}{Z^2} > 0$ from (22). The effect of $\tau$ on $\hat{k}$ is negative. Thus, urban wages decrease, whereas rural wages increase. Therefore, given the increasing wages and the reduced cost of raising children, an increased tax rate enhances the fertility rate in the rural area.

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