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COMPENSATION RULES AND INVESTMENT UNDER LAND-TAKING

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The purpose of this paper is to analyze the relationship between urban systems and compensation rules when government takes private land for public use. Numerous papers in *Law and Economics* have analyzed the problem of title transfer in land transactions. They do not, however, deal with land-taking or title transfer in the framework of spatial economics for simplification of the model. When government plans for provision of public goods, it often needs land in the region to do it. In the United States, for example, if public goods increase the utility level of each household in the city, the land required to provide them can be expropriated with fair compensation. In this paper, we focus on this compensation rule for land-taking and attempt to analyze the effects of it on a spatial model and the landowners' investment behavior on their own land.

1. Introduction

Land is often used as an input factor in the production of public goods such as roads, harbors, and parks. When neither households nor firms locate on the land, it is relatively easy to take it as an input factor. However, when households or firms locate on the land, the cost of taking it is often higher and the time required to obtain it increases. Therefore, when public projects can improve the welfare of households, the government may take the land from the point of view of public welfare. In the case of the United States this occurs in accordance with the U.S. Compulsory Purchase of Land Act.[1] If the land-taking is carried out in accordance with the Act, the households in question must select another residence in the urban area. The movement of households will change land rent, urban fringe, and utility level. Since a change in land rent affects all land owners' revenue, and because the existence of a threat of land-taking also affects investment behavior, it is important to discuss the issue of compensation for the land.

[1] In the United States, the Fifth and Fourteenth Amendments regulate land-taking.

When we consider government compensation for land-taking, we must take into account the land's title transfer. Compensation is based on two rules: property and liability.[2] The former establishes the title of the last rightful landowner. Thus, the current owner is at risk of losing the property to the last rightful landowner. On the other hand, the latter establishes the title of current landowner. In this case the last rightful landowner receives any monetary compensation. This system is similar to the Torrens System, which has been adopted by some states in the U.S. Thus, the change of land rents, urban fringe, and utility level under the property rule is different from that under the liability rule. In previous research of title transfer, Miceli, Sirmans, and Turnbull (1998) attempted to analyze title compensation and incentives to land use in the case where the claimant may claim his land's property. Blume, Rubinfeld, and Shapiro (1984) analyzed how the risk of land-taking affects the landowners' incentive to develop the land and showed that employing a non-compensation rule brings about efficient investment behavior when the government takes the land to provide public goods. Michelman (1967) argued for an adoption of unified standards when the government compensates landowners for the land taken.

Since the above studies mainly focused on the landowners' behavior, they did not analyze it from the viewpoint of spatial economics. As public goods such as parks and roads are provided in urban areas, it is necessary to analyze this problem within the framework of a spatial economic model. Lee and Fujita (1997) and Cho (1997) analyzed the provision of public goods theoretically, for example, in a greenbelt. Lee and Fujita (1997) discussed the optimal location of a greenbelt. According to them, when a greenbelt is treated as a pure public good, the optimal location of it should be the urban fringe. Conversely, when it is treated as an impure public good, its optimal location should be the inside of the urban area. Cho (1997) showed through an empirical analysis that the externality of congestion caused by the greenbelt could overstate land rent. Moreover, Lee and Hosoe (1999) analyzed the optimal provision level of greenbelt and showed that it is possible to result in urban sprawl when invoking the provision. The literature, however, did not consider the compensation rule for the land-taking in detail.

The purpose of this paper is to analyze the compensation problem for land-taking within the framework of spatial economics when the government provides a public good that requires land as an input factor. In particular, we focus on the compensation problem and attempt to analyze the effect of each compensation rule on the spatial system and the investment behavior of landowners. The basic model is presented in the next section. In section 3, we attempt to derive the market equilibriums and analyze the investment behavior of landowners. In section 4, we investigate the relationship between scale of land-taking and household utility. In the final section, we conclude our analysis and comment on remaining issues.

2. The model

We consider a linear city which has a Central Business District (CBD) at location *0* and plain land at each location. Population N resides in the city. We assume that N is given exogenously, that is, our spatial model is a closed city model. Each household residing in the city

[2] Since it is possible to mistake a transfer of the land title during registration, the rightful landowner is not necessarily the same as the current owner.

receives income Y and commutes from residence to the CBD, which has the only opportunity of employment in the city. Land at each location in the city is owned by absentee landowners, who purchase it for agricultural rent. They invest capital, k, in their land and get revenue from the determined market land rent based on investment cost and agricultural rent.

The time line of the model is as follows. In the first stage, an absentee landowner purchases land for agricultural rent. Second, the government, which is planning to provide a public good in the city, officially announces the land-taking plan. The landowner determines his or her investment level in the land under the compensation rule. Finally, after the government carries out the land-taking, households' behavior in the urban area determines the equilibrium land rent, with the landowner getting the revenue from such rent. After the land has been taken, the government uses land as an input factor to produce public goods and compensates landowners.

We assume that land is the only input factor to produce public goods and that the land from 0 to x_1 is taken by the government when land is required for such goods. The location of the public goods provided in the city is considered as given, from 0 to x_1. We also assume that the provision level is proportionate to the amount of land as an input factor. Thus, we define the provision level of public goods as follows.

$$G(x_1) = g\int_0^{x_1} L(x)dx, \qquad g > 0, \tag{1}$$

where x represents the distance from the CBD, and $L(x)$, the land distribution at x. Moreover, x_1 denotes the fringe of land that is used as an input factor to provide for public goods. We consider three compensation rules:

rule 1: No-compensation;
rule 2: r_a compensation; and
rule 3: market land rent compensation;

Under rule 1, the government does not compensate landowners. Needless to say, it can proceed with a land-taking without any cost. In the case of rule 2, the government compensates those landowners with the agricultural rent before their own investments for the land. Finally, in case of rule 3, the government compensates them with the rent that they would derive if the land-taking were not carried out.

2.1 Landowners' behavior

We assume that the land in the city is owned by absentee landowners, who purchase it at the price of agricultural rent and can increase the value of the land through investment in it.[3] This kind of investment is considered to have an influence on the household's utility. The land rent is determined on the basis of the value of the land, which is improved by the investment. We define the raw land rent after investment as follows.

$$r_a + ck(x), \tag{2}$$

[3] We can interpret the investment in this model as the development of residential sites.

where r_a is agricultural rent and $k(x)$ is the investment level at x. Equation (2) is a continuous function with respect to x, and $k' > 0$. c is the marginal cost for investment in the land.

2.2 Household behavior

The utility level of households depends on their consumption of composite goods, lot size, public goods provided in the city (if provided), and the investment level in land by the absentee landowner. Each household decides its consumption level of composite goods and lot size. The provision level of public goods and the investment level in land, however, are decided by the government and the landowners, respectively. We specify the utility function of households as follows:

$$u = \alpha \ln z + \beta \ln s + \gamma \ln(1 + G) + \sigma \ln k, \tag{3}$$

where z, s, G, and k represent consumption of composite goods, lot size, the provision level of public goods and the level of investment in land, respectively. We assume that $\alpha + \beta = 1$, $\gamma, \sigma > 0$. Each household must commute to the CBD to obtain income, Y. t and x represent the transportation cost and the distance from the CBD, respectively. We define the transportation cost function as follows:

$$T(x) = tx. \tag{4}$$

Each household maximizes its utility level subject to the budget constraint. The compensation to the landowner whose land is taken is paid by a lump-sum tax that the government imposes on households. Note that the budget constraint depends on the land-taking compensation rule. In the case of rule 1, the zero-compensation rule, the government does not compensate the landowner. The government does not have to impose a tax on each household. When we represent the market land rent under rule 1 as $R_1(x)$, each household faces the following budget constraint:

$$Y = z + R_1(x)s + tx. \tag{5}$$

Next, we consider the budget constraint under rule 2. The government compensates the landowner for the land-taking with agricultural rent, r_a. In this case, the tax per household is given by

$$\frac{1}{N} r_a \int_0^{x_1} L(x)dx. \tag{6}$$

If $R_2(x)$ is the market land rent at x under rule 2, the budget constraint that a household faces at x is as follows:

$$Y - \frac{1}{N} r_a \int_0^{x_1} L(x)dx = z + R_2(x)s + tx. \tag{7}$$

Finally, under rule 3, the government has to compensate landowners for the taking with the market land rent *R(x),* determined when the taking did not occur. In this case, the tax imposed on households in the city is

$$\frac{1}{N}\int_0^{x_1} R(x)L(x)dx. \tag{8}$$

Thus, the budget constraint of the household at x is as follows.

$$Y - \frac{1}{N}\int_0^{x_1} R(x)L(x)dx = z + R_3(x)s + tx, \tag{9}$$

where $R_3(x)$ is the market land rent under rule 3.

In the next section, we derive the equilibriums of the urban system under each compensation rule and landowners' optimal investment level.

3. Market equilibrium

In the previous section, we specified the utility function of a household that resides in the city with budget constraint under each compensation rule. We now investigate the equilibrium land use and investment to the land by landowners. It is useful to adopt the bid rent function approach to derive the equilibrium solutions. To begin, we discuss the case in which land-taking does not occur. We then derive equilibrium solutions under each compensation rule.

3.1 The case of no land-taking

If the government has no plan for public goods in the city, no landowners face the risk of a taking. Since the government does not have to finance for compensation, it would not impose on households a tax for it[4]. Each household's budget constraint in this case is given by

$$\mathrm{Y} = \mathrm{z} + \mathrm{R(x)s} + \mathrm{tx}. \tag{10}$$

We introduce the bid-rent function approach to derive the market equilibrium solutions. The bid-rent function under the no land-taking case is defined as follows:

[4] In this case, the public goods are not provided in the city. It means $G=0$.

$$R(x, u) = \max_{s,z}\left[\frac{\mathrm{Y - tx - z}}{\mathrm{s}} \quad \middle| \; u = \alpha \ln z + \beta \ln s + \sigma \ln k\right]. \tag{11}$$

In the maximization problem of (11), we may solve the utility constraint for u and obtain the equation of indifference curve as follows:

$$Z = s^{-\frac{\beta}{\alpha}} k^{-\frac{\sigma}{\alpha}} e^{\frac{u}{\alpha}}. \tag{12}$$

Substituting (12) into (11), we can deal with the maximization problem as unconstrained maximization, that is,

$$R(x, u) = \max_{s}\left[\frac{Y - tx - Z}{s}\right]. \tag{13}$$

The first order condition for the maximization problem is

$$\frac{\mathrm{Y - tx - Z}}{\mathrm{s}} = \frac{\partial \mathrm{Z}}{\partial \mathrm{s}} = \frac{\beta}{\alpha} s^{-1} Z. \tag{14}$$

Using (14), we can derive a household's consumption of composite goods as follows:

$$Z = \alpha(Y - tx). \tag{15}$$

A bid-max lot size can be determined by (12), (14) and (15), that is,

$$s(x, k, u) = \alpha^{-\frac{\alpha}{\beta}} k^{-\frac{\sigma}{\beta}} e^{\frac{u}{\beta}} (Y - tx)^{-\frac{\alpha}{\beta}}. \tag{16}$$

Thus, the bid-rent function is

$$R(x, k, u) = \alpha^{\frac{\alpha}{\beta}} \beta k^{\frac{\sigma}{\beta}} e^{\frac{-u}{\beta}} (Y - tx)^{\frac{1}{\beta}}. \tag{17}$$

Since landowners get their revenue only from the market land rent, they try to invest in their land to maximize revenue. As we assume that the marginal cost of investment in land is constant c, the landowners' problem is as follows:

$$\max_{k} R(x, k, u) - ck - r_a. \tag{18}$$

The first order condition for this problem is given by

$$\frac{\partial R(x, k, u)}{\partial k} - c = 0. \tag{19}$$

We can derive the revenue maximization investment level at x under u, $k(x,u)$ easily:

$$k(x, u) = \alpha^{\frac{\alpha}{\beta-\sigma}} c^{\frac{-\beta}{\beta-\sigma}} \sigma^{\frac{\beta}{\beta-\sigma}} e^{\frac{-u}{\beta-\sigma}} (Y - tx)^{\frac{1}{\beta-\sigma}}. \tag{20}$$

We set up the following assumption to satisfy the second order condition for maximization, that is, $\beta > \sigma$. Substituting (20) into (16) and (17), the bid rent and bid-max lot size functions can be rewritten by

$$R(x, u) = \alpha^{\frac{\alpha}{\beta-\sigma}} \beta c^{\frac{-\sigma}{\beta-\sigma}} \sigma^{\frac{\sigma}{\beta-\sigma}} e^{\frac{-u}{\beta-\sigma}} (Y - tx)^{\frac{1}{\beta-\sigma}}, \tag{21}$$

$$s(x, u) = \alpha^{\frac{-\alpha}{\beta-\sigma}} c^{\frac{\sigma}{\beta-\sigma}} \sigma^{\frac{-\sigma}{\beta-\sigma}} e^{\frac{u}{\beta-\sigma}} (Y - tx)^{\frac{-(\alpha+\sigma)}{\beta-\sigma}}. \tag{22}$$

Recall that our model is a linear city which has its CBD at 0. We assume that the amount of land supplied at x, $L(x)$, is unity, that is, $L(x) = 1$. Since the population N is an exogenous variable in a closed city model, the population constraint is as follows.

$$\int_0^{x_f} \frac{1}{s(x, u)} dx = N, \tag{23}$$

where x_f is the fringe of the city. Moreover, the following equation is satisfied at the fringe x_f in equilibrium:

$$R(x_f, u) = r_a + ck(x_f). \tag{24}$$

Using the above equations, we can derive the equilibrium urban fringe and the equilibrium utility level in the situation of no lnad-taking.

$$x_f = \frac{1}{t}\left[Y - \alpha^{-\alpha} c^{\sigma} e^{u} r_a^{\beta-\sigma} (\beta\sigma^{\frac{\sigma}{\beta-\sigma}} - \sigma^{\frac{\beta}{\beta-\sigma}})^{\sigma-\beta} \right], \tag{25}$$

$$e^{u} = Y\alpha^{\alpha} c^{-\sigma} \left[r_a (\beta\sigma^{\frac{\sigma}{\beta-\sigma}} - \sigma^{\frac{\beta}{\beta-\sigma}})^{\sigma-\beta} + \sigma^{\frac{-\sigma}{\beta-\sigma}} \frac{Nt}{\beta - \sigma} \right]^{\sigma-\beta}. \tag{26}$$

The equilibrium urban fringe, x_f, can be obtained by substituting (26) for (25).

3. 2 The land-taking case

We now attempt to analyze the equilibrium under each compensation rule. To begin with, we turn our attention to the equilibrium under the no compensation rule. Blume, Rubinfeld, and Shapiro (1984) conclude that no compensation results in the most efficient investment decisions by landowners. As mentioned above, however, they do not analyze this problem in a spatial model

When the government adopts the no compensation rule (rule 1), the budget constraint of a household is given by (5). The bid-rent maximization problem under this rule is as follows:

$$R_1(x, u_1) = \max_{s,z}\left[\frac{\mathrm{Y - tx - z}}{\mathrm{s}} \quad | \quad u_1 = \alpha \ln z + \beta \ln s + \gamma \ln(1+G) + \sigma \ln k\right]. \quad (27)$$

We can rewrite this as an unconstrained maximization problem:[5]

$$R_1(x, u_1) = \max_{s}\left[\frac{\mathrm{Y - tx - Z_1}}{\mathrm{s}}\right], \quad (28)$$

where, Z_1 is

$$Z_1 \equiv s^{\frac{-\beta}{\alpha}} k^{\frac{-\sigma}{\alpha}} (1+G)^{\frac{-\gamma}{\alpha}} e^{\frac{u_1}{\alpha}}. \quad (29)$$

The first order condition for this problem is given by

$$\frac{Y - tx - Z_1}{s} = -\frac{\partial Z_1}{\partial s} = \frac{\beta}{\alpha} s^{-1} Z_1. \quad (30)$$

The consumption of a household's composite goods is derived from (29) and (30), that is,

$$Z_1 = \alpha(Y - tx). \quad (31)$$

From (29) and (31), the bid-max lot size and the bid-rent can be obtained, respectively, as follows:

$$s_1(x, k, u_1) = Ae^{\frac{u_1}{\beta}} k^{\frac{-\sigma}{\beta}} (Y - tx)^{\frac{-\alpha}{\beta}} \quad (32)$$

$$R_1(x, k, u_1) = \beta A^{-1} e^{\frac{-u_1}{\beta}} k^{\frac{\sigma}{\beta}} (Y - tx)^{\frac{1}{\beta}}. \quad (33)$$

[5] Subscript 1 is attached to distinguish the equilibrium solution under rule 1 from those of the other case.

We define A as $A \equiv \alpha^{-\frac{\alpha}{\beta}}(1+G)^{-\frac{\gamma}{\beta}}$. Since landowners decide on their investment to maximize revenue from land rent, the maximization problem for a landowner is given by

$$\max_{k} R_1(x, k, u_1) - ck - r_a. \tag{34}$$

The first order condition for this problem is as follows:

$$\frac{\partial R_1(x, k, u_1)}{\partial k} - c = 0. \tag{35}$$

Solving the above equation, the optimal investment in land is derived as follows:

$$k_1(x, u_1) = \sigma^{\frac{\beta}{\beta-\sigma}} c^{-\frac{\beta}{\beta-\sigma}} A^{-\frac{\beta}{\beta-\sigma}} e^{-\frac{u_1}{\beta-\sigma}} (Y - tx)^{\frac{1}{\beta-\sigma}}. \tag{36}$$

In this situation, the land, which locates at [0, x_1], is used as an input factor in order to produce the public goods in the city. The population constraint and the fringe condition are then given by

$$\int_{x_1}^{x_x^1} \frac{1}{s_1(x, u_1)} dx = N, \tag{37}$$

$$R_1(x_f^1, u_1) = r_a + ck_1(x_f^1, u_1) \tag{38}$$

Using (37) and (38), we can derive the equilibrium urban fringe and the equilibrium utility level under the no compensation rule:

$$x_f^1 = \frac{1}{t}\left[Y - c^{\sigma} A^{\beta} e^{u_1} r_a^{\beta-\sigma} (\beta\sigma^{\frac{\sigma}{\beta-\sigma}} - \sigma^{\frac{\beta}{\beta-\sigma}})^{\sigma-\beta} \right], \tag{39}$$

$$e^{u_1} = (Y - tx_1)c^{-\sigma} A^{-\beta} \left[r_a(\beta\sigma^{\frac{\sigma}{\beta-\sigma}} - \sigma^{\frac{\beta}{\beta-\sigma}})^{\sigma-\beta} + \sigma^{-\frac{\sigma}{\beta-\sigma}} \frac{Nt}{\beta-\sigma} \right]^{\sigma-\beta} \tag{40}$$

Next, we analyze the case in which the government compensates landowners for a taking with agricultural rent (rule 2). The government imposes on households a lump sum tax to finance the monetary costs for compensation. The tax under this rule is given as follows:

$$B \equiv \frac{r_a}{N}\int_0^{x_1} L(x)dx = \frac{r_a x_1}{N}. \tag{41}$$

Since the budget constraint of households under this compensation rule is given by (7), the maximization problem for the rent is as follows:

$$R_2(x, u_2) = \max_{s,z}\left[\frac{\mathrm{Y - tx - B - z}}{\mathrm{s}} \mid u_2 = \alpha \ln z + \beta \ln s + \gamma \ln(1+G) + \sigma \ln k\right]. \tag{42}$$

We can obtain the equilibrium solutions in the same way as rule 1. To begin, bid-max lot size and bid rent function under rule 2 are:

$$s_2(x, k, u_2) = Ae^{\frac{u_2}{\beta}} k^{-\frac{\sigma}{\beta}} (Y - tx - B)^{-\frac{\alpha}{\beta}}, \tag{43}$$

$$R_2(x, k, u_2) = \beta A^{-1} e^{\frac{-u_2}{\beta}} k^{\frac{\sigma}{\beta}} (Y - tx - B)^{\frac{1}{\beta}}. \tag{44}$$

Landowners' revenue maximization investment under this rule is:

$$k_2(x, u_2) = \sigma^{\frac{\beta}{\beta-\sigma}} c^{-\frac{\beta}{\beta-\sigma}} A^{-\frac{\beta}{\beta-\sigma}} e^{-\frac{u_2}{\beta-\sigma}} (Y - tx - B)^{\frac{1}{\beta-\sigma}}. \tag{45}$$

The population constraint and the urban boundary condition are, respectively, as follows:

$$\int_{x_1}^{x_f^2} \frac{1}{s_2(x, u_2)} dx = N, \tag{46}$$

$$R_2(x_f^2, u_2) = r_a + ck_2(x_f^2, u_2). \tag{47}$$

Using these conditions, it is easy to derive the equilibrium for urban fringe and utility:

$$x_f^2 = \frac{1}{t}\left[Y - B - c^{\sigma} A^{\beta} e^{u_2} r_a^{\beta-\sigma} (\beta\sigma^{\frac{\sigma}{\beta-\sigma}} - \sigma^{\frac{\beta}{\beta-\sigma}})^{\sigma-\beta}\right], \tag{48}$$

$$e^{u_2} = (Y - B - tx_1)c^{-\sigma} A^{-\beta}\left[r_a(\beta\sigma^{\frac{\sigma}{\beta-\sigma}} - \sigma^{\frac{\beta}{\beta-\sigma}})^{\sigma-\beta} + \sigma^{-\frac{\sigma}{\beta-\sigma}} \frac{Nt}{\beta-\sigma}\right]^{\sigma-\beta}. \tag{49}$$

Finally, we analyze the case of rule 3, the case in which the government compensates landowners for a taking with market-value rent, which is determined when the taking has not occurred. Since the government imposes a tax on households to pay for the taking, the amount of imposed tax per household is given by

$$D \equiv \frac{1}{N}\int_0^{x_1} R(x)L(x)dx. \tag{50}$$

Since the budget constraint of households under rule 3 is given by (9), the bid-rent maximization problem is

$$R_3(x, u_3) = \max_{s,z}\left[\frac{\mathrm{Y - tx - D - z}}{\mathrm{s}} \mid \ u_3 = \alpha \ln z + \beta \ln s + \gamma\ \ln(1+G) + \sigma \ln k\right] \tag{51}$$

Thus, we can obtain the bid-max lot size and bid-rent function as

$$s_3(x, k, u_3) = Ae^{\frac{u_3}{\beta}} k^{-\frac{\sigma}{\beta}} (Y - tx - D)^{-\frac{\alpha}{\beta}}, \tag{52}$$

$$R_3(x, k, u_3) = \beta\, A^{-1} e^{\frac{-u_3}{\beta}} k^{\frac{\sigma}{\beta}} (Y - tx - D)^{\frac{1}{\beta}}. \tag{53}$$

The revenue maximization investment level is given by

$$k_3(x, u_3) = \sigma^{\frac{\beta}{\beta-\sigma}} c^{-\frac{\beta}{\beta-\sigma}} A^{-\frac{\beta}{\beta-\sigma}} e^{-\frac{u_2}{\beta-\sigma}} (Y - tx - D)^{\frac{1}{\beta-\sigma}}. \tag{54}$$

The population constraint and the urban boundary condition are as follows:

$$\int_{x_1}^{x_f^3} \frac{1}{s_3(x, u_3)} dx = N, \tag{55}$$

$$R_3(x_f^3, u_3) = r_a + ck_3(x_f^3, u_3). \tag{56}$$

Using (52), (53), (54) and the above constraints, we can derive the equilibrium utility level and the urban fringe, respectively, as follows:

$$x_f^3 = \frac{1}{t}\left[Y - D - c^{\sigma} A^{\beta} e^{u_3} r_a^{\beta-\sigma} \left(\beta\sigma^{\frac{\sigma}{\beta-\sigma}} - \sigma^{\frac{\beta}{\beta-\sigma}}\right)^{\sigma-\beta}\right], \tag{57}$$

$$e^{u_3} = (Y - D - tx_1)c^{-\sigma} A^{-\beta} \left[r_a(\beta\sigma^{\frac{\sigma}{\beta-\sigma}} - \sigma^{\frac{\beta}{\beta-\sigma}})^{\sigma-\beta} + \sigma^{-\frac{\sigma}{\beta-\sigma}} \frac{Nt}{\beta-\sigma} \right]^{\sigma-\beta} . \quad (58)$$

3.3 Comparison among equilibriums

We now compare the equilibriums and analyze their character under each compensation rule. To begin, we take the no land-taking case as a benchmark, using a numerical example.[6] In Figure 1, the solid line denotes the improved agricultural rent through investment, i.e., the sum of agricultural rent and investment cost. The broken line denotes the market land rent based on it. It can be easily shown that the level of investment in land decreases with respect to the distance from CBD. Moreover, the slope of the market land rent curve is steeper than that of the investment curve.

We next compare the levels of investment in land under each compensation rule (Figure 2). We see that the levels of investment in land with a taking are larger than those of the case without one. The reason is that the demand for land that is not taken by the government increases when a taking occurred in the city. As a result, the investment level in the land not expropriated by the government is relatively higher than that in the case a no taking. Comparing the equilibriums, it can be said that the level of investment under the no compensation rule is larger than that in any of the others. Since under rule 2 or rule 3 the government imposes on households a lump sum tax, each household's disposable income under rule 2 or rule 3 is smaller than that under rule 1. Moreover, the gap of investment level in land between rule 1 and rule 2 is small when agricultural rent is relatively low. Generally, the household's income has a positive relation to the size of the city, i.e., the urban fringe.

There is a possibility that the investment level under rule 3 is lower than that under no land-taking near the urban fringe when agricultural rent is relatively low. Even though agricultural rent is the same under all cases, if the scale of the expropriated land is too small, the level of investment in the neighborhood of the urban fringe under rule 3 is smaller than that under the case of no taking. Intuitively, the demand for land that is not expropriated increases when a taking occurs; the market land rent curve moves upward and the investment curve also moves upward. On the other hand, each household's disposable income decreases because the government collects tax to compensate landowners for a taking. The slope of market land rent becomes steep as household income decreases. These effects offset each other; when the scale of a taking is not large, the increase in demand for land is relatively small. As a result, the market land rent does not move upward dramatically (Figure 3).[7]

[6] The simulation is based on the following parameter specifications: $a = 0.5$, $\gamma = 0.3$, $\sigma = 0.2$, $g = 0.2$, $N = 100$, $Y = 50$, $x_1 = 5$, $r_a = 20$, $c = 1$.

[7] $r_a=10$, in Figure 3.

Figure 1. Investment and market land rent

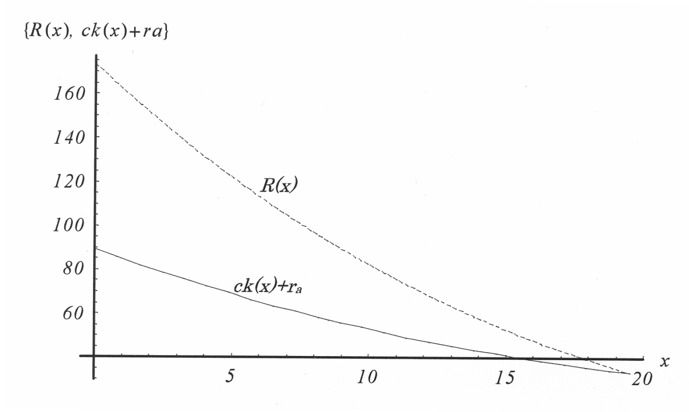


Figure 2. Investment under each compensation rules

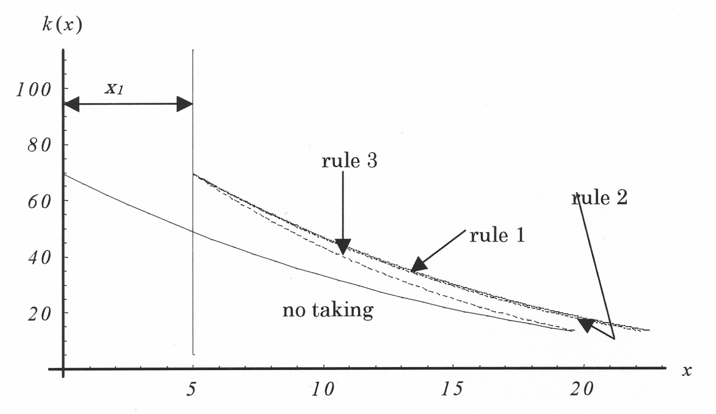
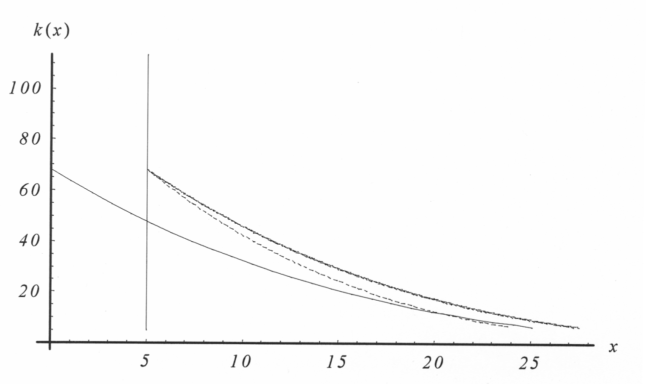


Figure 3. Investment under low agricultural rent



4. Land-taking and household utility

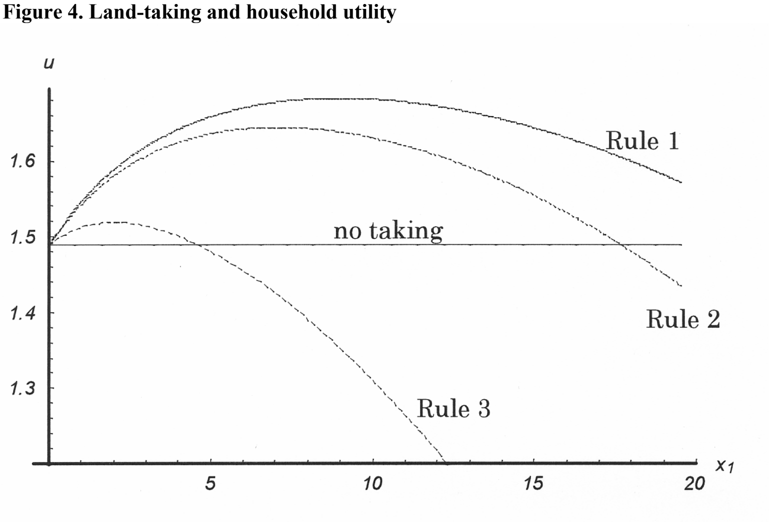
When the government plans the provision of public goods, it must take into consideration the influence on each household in the region. In particular, if land-taking were taken place for public goods, land rent would be raised because of the change in supply and demand in the land market. Furthermore, since the cost of compensation for taking land is financed by a lump-sum tax, household disposable income decreases. In this section, we investigate how a taking for public goods affects household utility and derive a utility maximization scale.

The equilibrium utility level under each compensation rule was obtained in equations (40), (49) and (58). We examine the relationship between land-taking scale and household utility, in the same manner before, i.e., numerical example. Figure 4 represents the relationship between land-taking scale x_1 and household utility u under a given value of g[8]. The solid line represents the utility level in the case of no taking. Dashed lines are the utility levels under rules 1, 2 and 3 from the top. In Figure 4, land-taking can raise the utility level under any compensation rule if the benefit from public goods is larger than the negative effects by the taking. However, when the taking scale exceeds a certain level, negative effects become high and utility decreases to a level lower than before the provision of the public goods.

Comparing the utility maximization scales under each compensation rule, the scale under rule 1 has the highest utility level. We investigate this case first. From (40), the necessary condition for the taking scale x_1 to maximize utility is:

$$\frac{-t}{Y - tx_1} + \frac{\gamma g}{1 + gx_1} = 0. \tag{59}$$

Figure 4. Land-taking and household utility



[8] $g=0.3$, in Figure 4.

If x_1 is positive, land-taking for public goods would raise household utility level. That is, the provision of public goods under rule 1 results in the positive effect exceeding the negative effect, and $x_{1,}$ makes household utility level the highest.

From (49), the necessary condition that maximizes household utility in the case of rule 2 is as follows:

$$\frac{-(r_a + Nt)}{NY - (r_a + Nt)x_1} + \frac{\gamma g}{1 + gx_1} = 0. \tag{60}$$

Similarly, if x_1 is positive, the provision of public goods can improve household utility level and the utility maximization scale is determined by (60). Moreover, from (59), (60), it can be explained that the land-taking scale of rule 1 is larger than that of rule 2 under the same level of g. This means that if the levels of benefit from public goods are the same, the size of public goods provision under rule 1 is larger than that under rule 2.

Next, we examine the case of rule 3. From (58), utility maximization scale x_1 must be satisfied following the necessary condition:

$$\frac{\gamma g}{1 + gx_1} + \frac{(1 + \beta - \sigma)Qt(Y - tx_1)^{\frac{1}{\beta - \sigma}} - t(\beta - \sigma)}{(\beta - \sigma)[(Y - tx_1) - Q(Y - tx_1)^{\frac{1+\beta-\sigma}{\beta-\sigma}} - QY^{\frac{1+\beta-\sigma}{\beta-\sigma}}]} = 0. \tag{61}$$

where Q is as follows:

$$Q = \frac{-1}{t}\alpha^{\frac{\alpha}{\beta-\alpha}}\beta c^{\frac{-\sigma}{\beta-\sigma}}\sigma^{\frac{\sigma}{\beta-\sigma}}e^{\frac{-u_3}{\beta-\sigma}}\frac{\beta - \sigma}{1 + \beta - \sigma}.$$

When the benefit from public goods is large enough, the determination of the land-taking scale depends on the population of the city, household income, transport cost, and agricultural rent. Table 1 represents the relationship between taking scale x_1 and each parameter under the three compensation rules.

The results of Table 1 can be explained with equations (26), (40), (49) and (58). Landowners' marginal cost of investment in land c does not influence the taking scale x_1. The change in c influences the utility by changing the landowner's investment behavior. However, such impact is constant and is not relevant to a taking. That is, the change of c simply moves the four utility lines in Figure 4 to an upward parallel and is irrelevant to the decision of x_1. In the case of rule 1, household disposable income would not change even though land-taking has taken place because of no compensation. From (26) and (40), therefore, the effect of the change in population N to utility level u or u_1 is the same and does not influence x_1. The same applies to agricultural rent r_a. In case of rule 2 and rule3, however, N and r_a are relevant to the burden of compensation cost. The increasing of N reduces compensation cost per capita; thus, x_1 becomes larger. Each equilibrium utility function shows that the higher the income Y, the larger the utility maximization scale. The higher the unit traffic cost t, the smaller the scale.

Table 1. Relation of the parameters to x_1

	Y	N	t	c	r_a
Rule 1	(+)	*	(-)	*	*
Rule 2	(+)	(+)	(-)	*	(-)
Rule 3	(+)	(+)	(-)	*	(-)

5. Conclusion

The problems of land-taking and title transfer have been analyzed in a simple case that involves the landowner and claimant in the model. Since a taking is a land transfer problem, we cannot ignore an analysis of this problem in a spatial system. One of the most interesting results is the fact that each equilibrium in a spatial model depends on the adopted compensation rule. The main purpose of this paper was the analysis of investment in land under each compensation rule. We used a simplified log-liner utility function to avoid making the model too complicated. In addition, we introduced the argument of a "law and economics" framework into a spatial system. We adopted the simplest model in urban economics, assuming, for example, that all households have identical preferences and that the city is a monocentric linear city. An analysis of the model in which these assumptions are formulated will be the subject of future research.

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